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In mathematics, the Krull dimension of a ring is the supremum of the lengths of chains of prime ideals in that ring. I think this could also be a nice way to think about complexity of thought. Thoughts that come up in an essay consist of chains of ideas which may or may not logically follow from one another. Perhaps we could refer to the dimension of an essay by the length of the longest chains of implication in ideas within the essay. Most of the stuff that chatGPT outputs would have essay dimension one; some of the better Paul Graham essays out there probably get to maybe five or six. At the long end of the spectrum would be mathematics writing itself, convoluted proofs being the most complex humanity could get to before the ideas become too intractable.

Given such a metric, would it be right to attempt deliberately thinking thoughts that go on and on? I don't think so. There's that old Thomas Mann quote about how the best ideas aren't forced in such a way, how the best writing just naturally flows forth. Of course, it is desirable to have ideas of nontrivial dimension. In fact, you could probably define a simpleton as someone who never has thoughts beyond dimension one. So how does one go about thinking such complex thoughts? It seems the only way to get idea-chains of dimension three or higher (dimension two is just your standard garden-variety implication/conditional) is to walk the space of thoughts in what you want to write about and to keep an eye out, then collect what you can. So the longer ideas appear more or less at random, but you're more likely to cultivate them the more you look.

Hang on, though. Some topics must be more prone to high-dimensional ideas than others. That's about half of what gets people to study things like mathematics: higher-dimensional ideas are pleasing to read about because they tell a story, and the more higher-dimensional ideas you're exposed to, the harder it is to be satisfied reading things that consist of simple one-dimensional observations. So you should be able to get better at thinking by just hanging around the ideas that are notorious for generating high-dimensional ideas.

We already know that higher-dimensional ideas are more interesting for the reader. As a writer, though, it may be more desirable to sprinkle the lengthier thought-chains amongst the simple observations. Humans may develop a taste for chasing implications, but to have one's writing solely consist of chain after chain seems just as drab as someone who can only think in singletons. There needs to be a pause for reflection or observation, an opportunity for the writer to change direction. Writing an essay is exciting because you're bound to come up with new ideas on the fly that you wouldn't have found by just passively thinking about a subject. Plugging on and on in the same direction keeps you from discovering things that are outside the trajectory of your current narrative.

So good writing (and by extension good thinking) contains idea-chains of length three or higher, remembers to pause for the opportunity to change direction, and takes place in spaces where lengthy idea-chains are more likely to be found. When your writing has long chains of ideas in it, it is more likely you are directly benefiting from the writing-exclusive thought process, because the paper or text editor helps hold ideas for you while you search for new ones. The essay reads as a queue, but is generated by a stack. Come to think of it, there could be multiple stacks, though as you add more and more stacks to your editor the harder it becomes to manage the overall flow of writing. Part of the editing process would include braiding the threads of ideas together so that what was once thought of with multiple idea stacks would then be manageable with just one. This is compassionate and keeps the reader in mind.

What does such compassion translate to in terms of having thoughts? Well, this is automatic if you write essays mainly for yourself. It's like commenting code well: you do it because one day, you might come

back to something you've written and have no idea what the hell it's trying to say. And reading your own writing is part of the feedback loop that leads to finding new ideas. It also helps other people who read your essays make coherent comments about your work. It might also be possible that in the process of finding things to cut that you find idea-chains that could be spliced together, things in two different paragraphs that work better in just one.

Going back to the idea of hanging around areas that are prone to generate interesting thoughts, one of the things I miss about translating the classics was that the ideas went on for as long as they needed to. Greek and Latin, perhaps partially because they are inflected languages with more tolerance for varying word order and hypotaxis, seem to be the natural medium for having lengthy yet substantial ideas. Some sentences in Cicero go on for five lines, to say nothing of writers such as Thucydides. It has been said that all of western philosophy consists of footnotes to Plato, and I think this is true beyond the observation that the Greeks grabbed most of the low hanging fruit in philosophical writing. And it must be good for one's brain to have complex thoughts idling even when not translating things directly. When I was a classics student, I tried to make an effort to not just translate things back into English without thinking about what gets lost when leaving Greek or Latin. Any classics student will confirm that the better you learn the language, the more you'll stumble into things that just sound awkward in English, but really do work in the language they were written in. So in the best case scenario, you read the Greek and then your subconscious mind then uses the Greek thoughts as monomers for having new thoughts instead of relying solely on thoughts thought of in English.

Unfortunately, I don't think it would be the most efficient thing to collect complex languages in order to have exotic and intricate thoughts. There's an upfront cost to introducing the material to your brain, and then there's a maintenance cost in constantly having to look up words. Of course, there does come a point where you pick things up faster and faster having attained a critical mass of language knowledge, but still... mastery takes time. Arguably one could find solace in having changed one's own state of mind by merely being exposed to anything in a novel language. I certainly feel like more of an athlete having run a single marathon, even if I never manage to do it again.

Are higher-dimensional thoughts merely an amusement? I think they could also contribute to the health of one's mind. We often speak of mental health in mentioning illness and deficiency, but just as there is a difference between someone who isn't suffering from disease and someone who is in good physical shape, having more complex ideas is a step above being without anxiety, depression, or something similar. It certainly feels easy to read simple writing having navigated more complex idea-bodies in, say, a math textbook. This is not to say, though, that I am motivated by keeping my brain in shape. I will, for better or worse, see higher-dimensional thoughts first and foremost as pure pleasure. I know of no other way to think.